One Dimensional Simulation for Peltier Current Leads

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Abstract—**Current leads, which connect superconducting magnets at the liquid helium temperature and power supplies at the room temperature, are the major source of heat leaking into cryostats, and therefore largely determines the running cost of magnet systems. Heat leak can be reduced by using high-temperature superconductors as the low-temperature (4 K–77 K) segments of current leads. Another method to reduce heat leak, recently proposed by one of us (S. Y.), uses Peltier thermoelectric elements as the high-temperature (200 K–300 K) segments of current leads. These thermoelements effectively pump heat out of cryostats without using separate sources of electricity. We carried out experiments and numerical calculations with such Peltier current leads and found out that they reduce heat leak at 77 K by 20–30 percent.**

I. Introduction

In 1834 Peltier observed that heat is absorbed or generated when an electric current crosses a junction between two different materials. This phenomenon, called the Peltier effect, has been used to build silent and reliable refrigerators. [1]

Most thermoelements (materials that exhibit large thermoelectric effect) manufactured today are alloys of bismuth (Bi), tellurium (Te), selenium (Se), and antimony (Sb), heavily doped to create either an excess (n-type) or a deficiency (p-type) of electrons. When such a thermoelement is used as part of an electric circuit, it pumps heat from one junction to the other. The direction of heat pumping is parallel to the current for n-type elements, and antiparallel for p-type elements (Fig. 1(a)). Usually they are used in pairs as in Fig. 1(b) for refrigeration.

One of us (S. Y.) recently observed [2] that thermoelements can be used to pump heat out of cryostats for superconducting magnets as shown in Fig. 1(c). Since the electric current that drives the superconducting magnet itself is used to cool the magnet, there is no need for separate sources of electricity.

Here we present experimental, analytical, and computer simulation results of such Peltier current leads.

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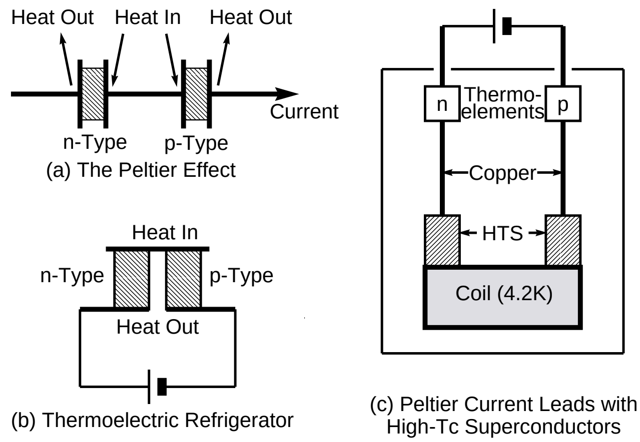

Fig. 1. The Peltier effect, a Peltier refrigerator, and Peltier current leads

II. The Peltier Effect

Let T_1 and T_2 be the temperatures of the cold and hot sides of a thermoelement, respectively (see Fig. 2). Then the heat absorbed at the cold side is

$$Q = \alpha T_1 I - \tfrac{1}{2} I^2 R - K(T_2 - T_1), \tag{1}$$

where I is the electric current through the element, and the three quantities α, R, and K characterize the thermoelement: α is the Seebeck coefficient, R the electric resistance, and K the thermal conductance.

In terms of the "shape factor" $\zeta = IL/A$ where L and A are the length and the cross-sectional area of the element, respectively, we can rewrite (1) as

$$Q/I = \alpha T_1 - \rho\zeta/2 - \kappa(T_2 - T_1)/\zeta, \tag{2}$$

where ρ and κ are the resistivity and the thermal conductivity of the element, respectively.

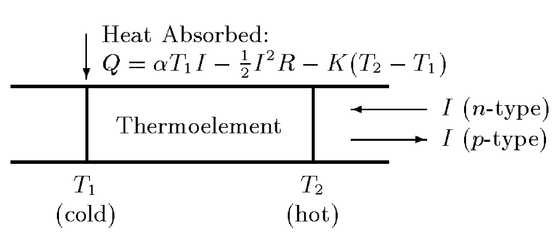

Fig. 2. Calculation of pumped heat.

Typical characteristics for bismuth-telluride thermoelements are

$$\left.\begin{array}{l} \alpha = 2.0 \times 10^{-4}\,\mathrm{V\,K^{-1}} \\ \rho = 1.0 \times 10^{-5}\,\Omega\,\mathrm{m} \\ \kappa = 1.5 \qquad \mathrm{W\,m^{-1}K^{-1}} \end{array}\right\} \tag{3}$$

near $T_2 = 300\,\mathrm{K}$. (The product $\rho\kappa$ is only twice as much as what the Wiedemann-Franz law (7) gives for 300 K.)

III. The Conductor Lead

One-dimensional energy balance of a general conductor lead is

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - H + \frac{I^2 r}{A} = 0, \tag{4}$$

where k is the thermal conductivity, A the cross-sectional area, r the electric resistivity of the conductor, T the temperature, I the current, and H the rate of heat transfer to the coolant.

For a gas-cooled current lead with good thermal conduction between the lead and gas, the heat transfer H can be written as $\dot{m}C_p dT/dx$ to a good approximation [3], where $\dot{m}$ is the flow rate (mass per unit time) and C_p is the specific heat at constant pressure of the gas.

With this approximation, and by making the substitution $dz = Idx/(kA)$, we can write (4) as

$$\frac{d^2T}{dz^2} - \frac{\dot{m}}{I}C_p\frac{dT}{dz} + kr = 0. \tag{5}$$

One important class of gas-cooled current leads is defined by *the self-cooling condition*, under which the heat leak at the cold end of the conductor, namely

$$Q_0 = kA(dT/dx)_{x=0} = I(dT/dz)_{z=0},$$

is used to evaporate liquefied gas, and the evolved gas is then used to cool the conductor lead. Under this condition, gas flow rate is

$$\dot{m} = Q_0/C_L, \tag{6}$$

where C_L is the latent heat of evaporation of the gas. Although there is nothing sacred about the self-cooling condition, we shall use it to determine a typical value for $\dot{m}$ other than $\dot{m} = 0$.

We can solve (4) in general and (5) in particular by any good integration routine.

IV. Experiment and Simulation

To see how these equations for thermoelements and conductor leads are applicable in the real world, we carried out experiments with the setting shown in Fig. 3. A U-shaped copper rod (diameter = 16 mm, $RRR = 100$) was dipped in liquid nitrogen (77 K), and currents were applied ($I = 0$ to 700 A). Temperatures just above and below the thermoelements were measured. The thermoelements we used were bismuth-telluride based ($A = 28\,\mathrm{mm} \times 28\,\mathrm{mm}$, $L = 5\,\mathrm{mm}$). The length of the copper rod above liquid nitrogen was set to 245 mm or 734 mm.

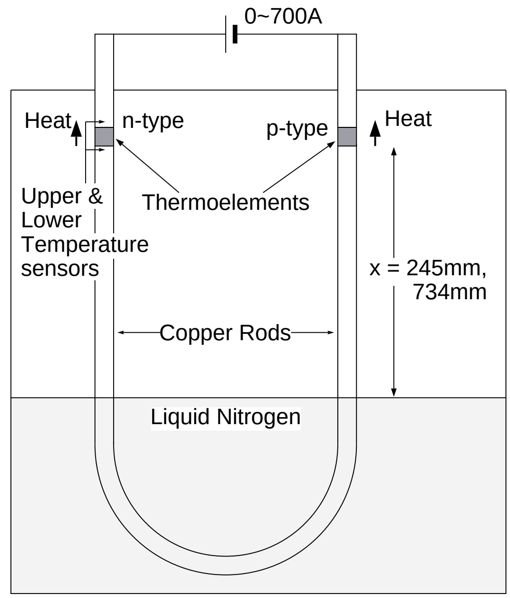

Fig. 3. Experimental Setup

In Fig. 4, the marks show the measured upper (hot side) and lower (cold side) temperatures of the n-type thermoelement; the p-type element gave similar results.

On the basis of the measured upper temperatures, we calculated lower temperatures (solid lines in Fig. 4) using equations (1)[1] and (4) with $H = 0$. As can be seen in Fig. 4, they are in good agreement with the measured lower temperatures.

[1] In this calculation we used the values Komatsu Electronics Inc. supplied us: $\alpha_p = 1.91 \times 10^{-4}\,\mathrm{VK^{-1}}$, $\alpha_n = 2.05 \times 10^{-4}\,\mathrm{VK^{-1}}$, $\rho_p = 0.99 \times 10^{-3}\,\Omega\,\mathrm{cm}$, $\rho_n = 0.97 \times 10^{-3}\,\Omega\,\mathrm{cm}$, $\kappa_p = 1.50 \times 10^{-2}\,\mathrm{W\,cm^{-1}K^{-1}}$, $\kappa_n = 1.65 \times 10^{-2}\,\mathrm{W\,cm^{-1}K^{-1}}$.

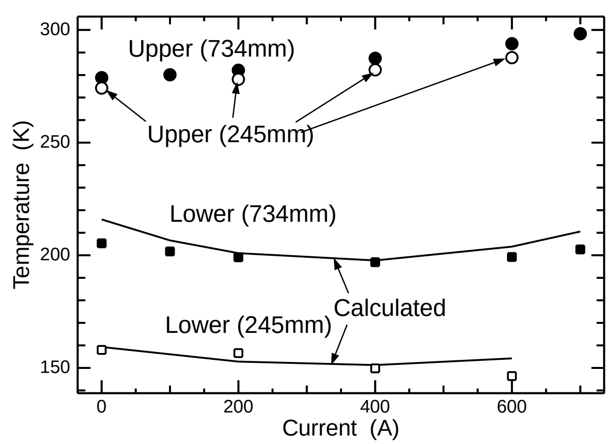

Fig. 4. Comparison of Experiment and Simulation

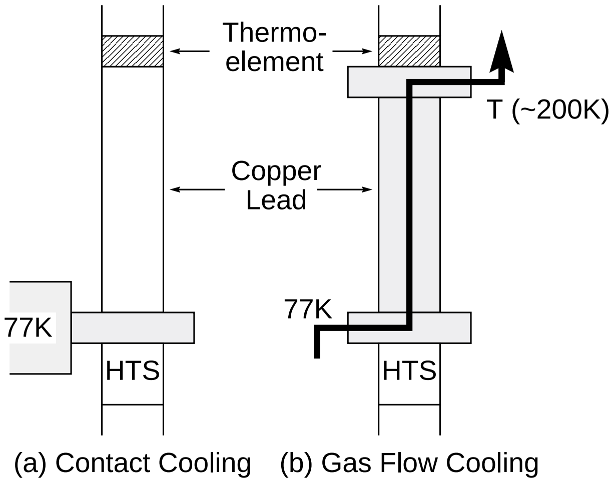

Fig. 5. Cooling methods

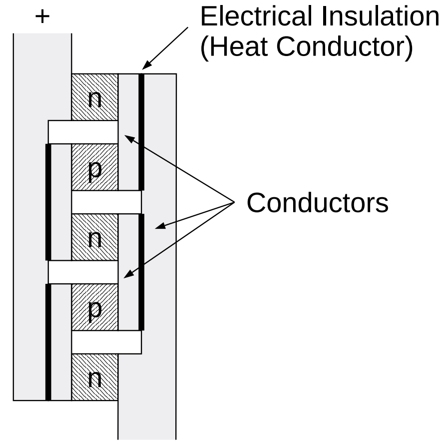

Fig. 6. Connecting thermoelements electrically in series, thermally in parallel.

V. High-Temperature Superconductors

High-T_c superconductors (HTS) generally have low heat conductivity, and therefore serve as good current leads under liquid nitrogen temperature [4]. If we use sufficiently long HTS, heat leak into liquid helium temperature can be arbitrarily reduced.

HTS are used as the low-temperature (e.g. 4 K–77 K) segments of current leads, whereas thermoelements are more effective near room temperature (e.g. 200 K–300 K). In between (77 K–200 K) we use conventional copper leads as shown in Figs. 1(c) and 5.

VI. Analytical Results

In simple cases we can analytically solve the relevant equations. For example, in the equation for the conductor lead (5) we assume that $\dot{m} = 0$, which corresponds to the contact-cooling case (Fig. 5(a)), and apply the Wiedemann-Franz law

$$kr = L_0 T, \quad L_0 \approx 2.45 \times 10^{-8}\,\mathrm{W\,\Omega\,K^{-2}}. \qquad (7)$$

For the boundary condition we simply let $T = 0$ at $z = 0$. Then the solution of (5) is

$$T = C \sin \beta z, \qquad \beta = \sqrt{L_0},$$

and heat leak per unit current at the cold end of the conductor lead ($z = 0$) is

$$Q_0/I = (dT/dz)_{z=0} = C\beta,$$

which is to be minimized.

At the hot end of the conductor lead ($z = z_1$), we have two more boundary conditions (see (2)),

$$C \sin \beta z_1 = T_1$$

and

$$(dT/dz)_{z=z_1} = C\beta \cos \beta z_1 = D - ET_1,$$

where

$$D = \rho\zeta/2 + \kappa T_2/\zeta, \quad E = \alpha + \kappa/\zeta.$$

These can be easily solved for the minimum of $C\beta$,

$$(C\beta)_{\min} = \beta D/\sqrt{\beta^2 + E^2}. \qquad (8)$$

The optimum temperature at the thermoelement-conductor junction is

$$(T_1)_{\min} = DE/(\beta^2 + E^2).$$

Heat from the thermoelement to the conductor is positive:

$$D - E(T_1)_{\min} = \beta^2 D/(\beta^2 + E^2) > 0.$$

Using the typical values given in (3), we have $(C\beta)_{\min} = 0.034\,\mathrm{W/A}$ when $\zeta = 5200\,\mathrm{A/m}$.

According to (8), to reduce heat leak we have to increase the Seebeck coefficient α while keeping ρ small.

One might think that it helps to use n thermoelements electrically in series and thermally in parallel as in Fig. 6, but this is not true, for the effect is equivalent to multiplying the three quantities α, ρ, and κ by n, and hence equivalent to multiplying both D and E by n. Equation (8) shows that the heat leak actually increases if we increase n.

VII. Numerical Simulation

We carried out numerical calculations for a system consisting of an HTS (which spans the temperature range from 4 K to 77 K), a copper lead (77 K to T), and a thermoelement (T to 300 K). The sizes of the copper lead and the thermoelement (and hence the junction temperature T) are chosen so as to minimize heat leak at the 77 K end of the HTS. Since there are many variables over which to optimize, design of algorithm is important. What follows is the algorithm we used.

First, we specify a value for dT/dz at the cold end of the conductor lead ($z = 0$). Then, beginning with this

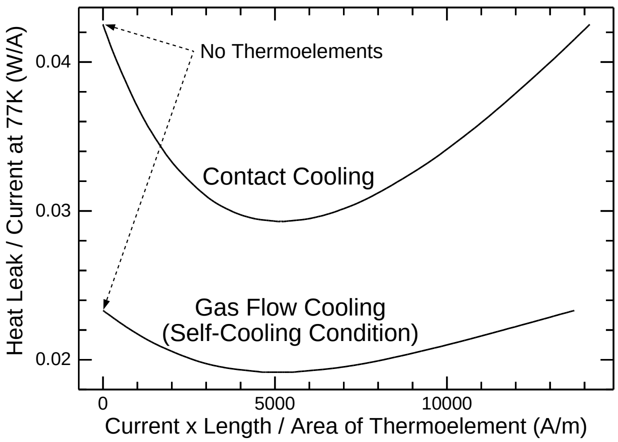

Fig. 7. Heat leak vs. "shape factor" of thermoelement

value of $(dT/dz)_{z=0}$ and $T_{z=0} = 77\,\mathrm{K}$, we numerically integrate (4) in the positive z direction. At each step of the integration (i.e. $z \leftarrow z + \Delta z$), we get a new triplet $(z, T, dT/dz)$. We then substitute the values of T and dT/dz into T_1 and $-Q/I$ of the Peltier equation (2),

$$-dT/dz = \alpha T - \rho\zeta/2 - \kappa(300\,\mathrm{K} - T)/\zeta,$$

and solve this quadratic equation in ζ,

$$\zeta_\pm = \frac{(dT/dz + \alpha T) \pm \sqrt{(dT/dz + \alpha T)^2 - 2\rho\kappa(T_2 - T)}}{\rho}.$$

We proceed with such integration steps, remembering only the maximum value of ζ_+ and the mimimum value of ζ_- so far, until the temperature T either exceeds 300 K or begins to decrease. When the integration is finished, we have a particular instance of one-to-two correspondence:

$$(dT/dz)_{z=0} \to (\zeta_{+\max}, \zeta_{-\min}).$$

We gather such instances of correspondence for sufficiently many values of $(dT/dz)_{z=0}$. Finally we plot $(dT/dz)_{z=0}$ (which is equal to Q_0/I) against $\zeta_{+\max}$ and $\zeta_{-\min}$. This simple algorithm gives the correct dependence on ζ of the smallest possible heat leak Q_0/I (optimized over z and hence over T), provided the dependence is unimodal.

Fig. 7 shows the results of this algorithm for

- the copper lead cooled at the bottom by a 77 K refrigerator as in Fig. 5(a), and
- the copper lead cooled by nitrogen gas flow from bottom (77 K) to top (T) as in Fig. 5(b) with nitrogen flow rate $\dot{m}$ given by the self-cooling condition (6).

We used standard routines for copper properties (with $RRR = 100$), helium and nitrogen properties, and the typical characteristics of thermoelements (3).

These graphs show that the optimum values for heat leak per current are

- 0.0293 W/A for a contact-cooled lead, and
- 0.0192 W/A for a gas-flow-cooled lead.

Also, if we look at the "shape factor = 0" points at the leftmost ends of the graphs, we see that without thermoelements, heat leak per current would be

- 0.0425 W/A for a contact-cooled lead, and
- 0.0233 W/A for a gas-flow-cooled lead.

In short, thermoelements reduce heat leak at 77 K by 31% and 18% for contact-cooling and gas-flow-cooling cases, respectively.

In the above we minimized heat leak at 77 K. We might minimize the sum of room-temperature refrigerator power and the power dissipated by ohmic heating in the copper/thermoelement lead. This requires knowledge of refrigerator efficiency, but if we take the ideal value, then the quantity to be minimized is

$$\text{heat leak} \times \tfrac{300-77}{77} + \text{ohmic loss within copper} + \text{ohmic loss within thermoelement.}$$

In the contact cooling case, the minimum of this quantity is 0.139 W/A, which is 16% smaller than the no-thermoelement value,

$$0.0425 \times \tfrac{300-77}{77} + 0.0425 = 0.165 \quad (\mathrm{W/A}).$$

Note that the factor $\frac{300-77}{77}$ refers to the ideal reversible refrigerator. If we use more realistic factor, then the reduction of power due to thermoelements will be somewhere between 16% and 31%, probably nearer to 31%. Similarly, in the gas flow cooling case, the reduction will be between 13% and 18%.

VIII. Conclusion and Discussion

We described how thermoelements can pump heat out of superconducting magnet systems. We showed by analytical and numerical calculations that such Peltier current leads are indeed possible.

We used a thermoelement only at the room-temperature end of the lead, because bismuth-telluride elements now in wide use are less efficient at lower temperatures. Other materials such as bismuth-antimony alloys, however, remain quite good at low temperatures; they tend to be even better in transverse magnetic fields. This interesting fact suggests that such elements be used in the magnetic field of the magnet they drive. A related phenomenon which may be utilized is the Ettingshausen effect, which pumps heat proportional to $\boldsymbol{H} \times \boldsymbol{j}$, where $\boldsymbol{H}$ is the magnetic field and $\boldsymbol{j}$ is the current density.

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